

Lecture Jan 28

Monday, January 28, 2019 11:05 AM

Induction

$p: \mathbb{N} \rightarrow \{\text{T}, \text{F}\}$ Prove $p(0)$ Base case

Also, prove $\forall n \in \mathbb{N}, p(n) \text{ IMPLIES } p(n+1)$. Generalization.

1. Let n be arbitrary in \mathbb{N} .

2. Assume $p(n)$

3 (actual proof)

4 $p(n+1)$

5 $p(n) \text{ IMPLIES } p(n+1)$ direct proof 2.4

6 $\forall n \in \mathbb{N}, p(n) \text{ IMPLIES } p(n+1)$ generalization 1.6

Example: Theorem: Consider any square "chessboard" with side length 2^n

If one square is removed, then the board can be filled

by L-shaped tiles (). Tiling: no gaps or overlaps.

Proof. Let $p(n) =$ "any $2^n \times 2^n$ chessboard with 1 square removed
can be filled with L-tiles."

Let $C_n =$ set of all $2^n \times 2^n$ chessboard with 1 square removed

$P(n) = \forall c \in C_n, c \text{ can be tiled}$

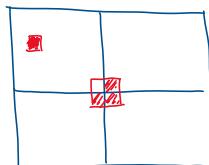
• $p(0)$ trivial.

• Let $n \in \mathbb{N}$ be arbitrary.

Let $c \in C_{n+1}$ be arbitrary

Divide c into 4 quadrants.

One quadrant contains a tile removed



c can be tiled.

$p(n+1)$ by generalization

$\forall n \in \mathbb{N}, p(n) \text{ IMPLIES } p(n+1)$ by generalization

Theorem 2 (corollary): Theorem 1 except square removed from

1.7.7

Theorem 2 (corollary). Theorem 1 except square removed from middle.

Theorem 3. For all $n \in \mathbb{N}$, $n \geq 3$ IMPLIES $q(n)$

Proof: For $n \in \mathbb{N}$, let $q(n)$: " $2n+1 \leq 2^n$ "

- Base case: $q(3)$ trivial. Why start at $q(3)$?
- Let $n \in \mathbb{M} = \{x \mid x \in \mathbb{N}, x \geq 3\}$ be arbitrary.

Assume $p(n)$

⋮
 $p(n+1)$

$p(n)$ IMPLIES $p(n+1)$

$\forall n \in \mathbb{M}, p(n)$ IMPLIES $p(n+1)$

$p(n)$

direct proof

generalization

induction

Let $p(n) = q(n+3)$ for $n \in \mathbb{N}$.

Can begin at $p(0)$. Equivalent to base case $q(3)$.

Just consider a different

Another way: $r(n) = "n \geq 3 \text{ IMPLIES } q(n)"$ where $n \in \mathbb{N}$

Base case: $r(0)$ vacuously true

Induction case:

- Prove by cases.
 - $n < 1$ vacuous
 - $n = 2$ "real" base case show $q(3)$
 - $n > 2$ induction step