

# Lecture Mar 11

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Algorithm search array  $A$  for key  $k$

Preconditions:  $A$  is indexed starting at 1 (optional),  $k$  can be checked for equality.

Postconditions: If key  $k$  is in  $A$ : return first index  $i$  such that  $A[i] = k$

If  $k$  not in  $A$ , return -1

$A, k$  are not modified.

Algorithm: Binary Search

Preconditions:  $A$  indexed starting at 1

$A$  sorted in nondecreasing order. If  $1 \leq i < j \leq \text{length}(A)$ , then  $A[i] \leq A[j]$ .

Elements of  $A$  and  $k$  are members of a totally ordered domain.

Algorithm: Sorting Array

Preconditions: elements of  $A$  from totally ordered set

Postconditions: elements of  $A$  after algorithm are a permutation of elements before,

$A$  non decreasing.

Algorithm: Merging arrays  $A$  and  $B$  to produce  $C$ .

Preconditions:  $A, B$  from totally ordered set

$A, B$  non decreasing

Postconditions:  $C$  non decreasing

$C$  permutation of  $A \cdot B$  (concat)

$A, B$  unchanged

Prove Correctness of Merge SortMERGE SORT( $A, n$ ):

- 1 if  $n > 1$
- 2 then:  $m \leftarrow \lfloor n/2 \rfloor$
- 3  $u \leftarrow A[1..m]$
- 4  $v \leftarrow A[m+1..n]$
- 5 MERGESORT( $u, m$ )
- 6 MERGESORT( $v, n-m$ )
- 7  $A \leftarrow \text{MERGE}(u, v)$

Additional precondition:  $A$  indexed 1 to  $n$ Proof: For  $n \in \mathbb{N}$ , Let $P(n)$ : "For all arrays  $A[1..n]$  with elements from a totally ordered domain. (Precondition) $P(n)$ : "If MERGESORT( $A, n$ ) is performed, then it eventually halts. At which time  $A$  is sorted in non-decreasing order, and the multiset of elements in  $A$  is unchanged" (Postcondition)

Proof by induction.

Let  $n \in \mathbb{N}$  be arbitrary.Let  $A[1..n]$  be an array of elements from a totally ordered domain(Base Case)  $n = 0, 1$  Test on L1 fails so  $A$  unchanged. $A$  sorted in non-decreasing order.Since  $A$  is arbitrary,  $P(n)$  true(Inductive Step) Let  $n > 1$  be arbitrary. Assume  $P(n')$  for all  $n' \in \mathbb{N}$  such that  $n' \leq n$ .Let  $A[1..n]$  be an arbitrary array with elements from a totally ordered domain.Test on L1 succeeds.  $m = \lfloor n/2 \rfloor$  from L2 So  $1 \leq m, n-m < n$ .By induction hypothesis, after L5, L6.  $u, v$  non-decreasing. and eventually halts.multiset of  $u$  is same as  $A[1..m]$ multiset of  $v$  is same as  $A[m+1..n]$ .The union of the multiset of elements in  $u$  and  $v$  is the multiset of elements originally from  $A$ .After L7,  $A$  is

- non-decreasing
- permutation of  $u, v$ .

• permutation of u·v.

Also, A eventually return

Since A is arbitrary, P(n) is true

By induction,  $\forall n \in \mathbb{N}, P(n)$ . Hence, MERGESORT is correct

### Proof of correctness for quicksort

QUICKSORT(A)

If  $|A| > 1$  then

pivot  $\leftarrow A[1]$

partition A into multisets L, E, G.

where L: less than pivot

E: equal to pivot

G: greater than pivot

Let  $P(n)$ : For all arrays A with n elements from totally ordered domain

If QS(A) performed, then

- eventually halts

- A sorted nondecreasing.

- multiset unaltered

Need to prove  $\forall n \in \mathbb{N} P(n)$  by induction

QUICKSORT(L)

QUICKSORT(G)

A  $\leftarrow L \cdot E \cdot G$