

# Lecture Mar 27

Wednesday, March 27, 2019 11:12 AM

## Closure Results

Suppose  $L_1 = \mathcal{L}(M_1) \subseteq \Sigma^*$  and  $L_2 = \mathcal{L}(M_2) \subseteq \Sigma^*$  where

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$  and  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$  are FA.

Then the following languages are accepted by FA.

$\Sigma^* - L_1, L_1 \cap L_2, L_1 \cup L_2, L_1 \cdot L_2, L_1^*, L_1^\dagger$

- Complement:  $(\Sigma^* - L_1)$  Flip the final states

$$M'_1 = (Q_1, \Sigma, \delta_1, q_1, Q_1 - F_1) \text{ and } \mathcal{L}(M'_1) = \Sigma^* - \mathcal{L}(M_1)$$

Proof Let  $x \in \Sigma^*$ . Then  $x \in \mathcal{L}(M_1)$

iff  $\delta^*(q_1, x) \in Q_1 - F_1$

iff  $\delta^*(q_1, x) \notin F_1$

Might not be true in NFAs  $\rightarrow$  iff

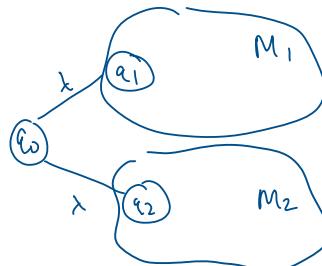
$x \notin \mathcal{L}(M_1)$

- Union

Proof:  $\lambda$ -transitions

$$M = (Q_1 \cup Q_2 \cup \{q_0\}, \delta, q_0, F_1 \cup F_2)$$

$$\delta(q_0, \lambda) = \{q_1, q_2\}$$



$$\text{Then } \mathcal{L}(M) = \mathcal{L}(M_1) \cup \mathcal{L}(M_2)$$

- Intersection Idea: Construct new DFA where each state is an ordered pair of  $M_1, M_2$

$$\text{Let } M = (Q_1 \times Q_2, \Sigma, \delta, (q_1, q_2), F_1 \times F_2)$$

$$\delta((q_1, q_2), a) = (\delta(q_1, a), \delta(q_2, a)) \text{ where } (p_1, p_2) \in Q_1 \times Q_2, a \in \Sigma.$$

$$\text{Then } \mathcal{L}(M) = \mathcal{L}(M_1) \cap \mathcal{L}(M_2).$$

- Concatenation Idea:  $\lambda$ -transitions from  $F_1$  to  $q_2$

$$\text{Let } M = (Q_1 \cup Q_2, \Sigma, \delta, q_1, F_2)$$

- Concatenation : Idea:  $\lambda$ -Transitions from  $q_1$  to  $q_2$

Let  $M = (Q_1 \cup Q_2, \Sigma, \delta, q_1, F_2)$

$$\delta(q, a) = \begin{cases} \{\delta_2(q, a)\} & \text{if } q \in Q_2 \\ \{\delta_1(q, a)\} & \text{if } q \in Q_1 \end{cases}$$

$$\delta(q, \lambda) = \{q_2\} \quad \text{if } q \in F_1$$

- $L_1^+$  Idea: Connect final state to starting states

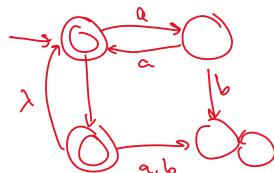
Let  $M = (Q_1, \Sigma, \delta_1, q_0, F_1)$

$$\delta(q, \lambda) = \{q_0\} \quad \text{if } q \in F_1$$

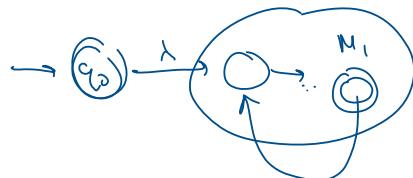
$$\delta(q, a) = \delta_1(q, a)$$

- $L_1^*$  Idea: Construct starting state with  $\lambda$ -transition

Note: Cannot make initial state final.



In this case, aa is also accepted.



Corollary Every regular language (i.e. L(R) for every regex R) can be accepted by a FA (i.e. exists FA M such that  $L(M) = L(R)$ ).

The converse is also true

Other closure results

- Reversal : For any string  $w \in \Sigma^*$ ,  $w^R = w^{REV}$  is the reversal of w  
Define  $L^{REV} = \{w^{REV} \mid w \in L\}$

Theorem: If L is regular, then  $L^{REV}$  is regular.

Swap initial states and final states. Reverse transitions

Let  $M' = (Q, \Sigma, \delta', F, \{q_0\})$

Swap initial states and final states. Reverse transitions.

Let  $M' = (Q, \Sigma, \delta', F, \{q_0\})$

Let  $x \in L(M')$  and suppose  $|y| = n > 0$   $y = x^{\text{REV}}$

There exists  $q_n \in F$  and  $q_{n-1}, \dots, q_0 \in Q$ . such that

$$q_{n-i} = \delta'(q_i, y_{n-i}) \quad \text{for } i = 0, \dots, n$$

Then, by def. of  $\delta'$ ,  $\delta(q_{i-1}, x_i) = q_i$

$$q_n \in \delta^*(q_0, x_1 \dots x_n) \quad x_1 \dots x_n \in L(M)$$

$$(y_1 \dots y_n)^{\text{REV}} \in L(M) \quad \text{so } y \in L(M)^{\text{REV}}$$