

Lecture Jan 30

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Induction: $q(n)$ true for even natural numbers.

Let $p(n) = q(2n)$

Then $\forall k \in \mathbb{N}$, $p(n)$ is the same as

$\forall n \in \mathbb{N}$. (n even) IMPLIES $q(n)$

Base case: $p(0) = q(0)$.

Induction step: $p(n)$ IMPLIES $p(n+1)$, same as
 $q(2n)$ IMPLIES $q(2n+2)$.

Although it is sufficient to prove $q(0)$ AND $\forall n \in \mathbb{N}$. $q(n)$ IMP $q(n+2)$.

but it might not be always true.

e.g. $q(0)$ IMP $q(2)$ works but

$q(1)$ IMP. $q(3)$ might not

AM/ GM Inequality.

Theorem: For all positive integers n and real numbers

a_1, a_2, \dots, a_n :

$$\left(\prod_{i=1}^n a_i \right)^{\frac{1}{n}} \leq \frac{1}{n} \sum_{i=1}^n a_i$$

Proof: Let $p(n)$ denote the predicate:

"For all real numbers $a_1, a_2, \dots, a_n \in \mathbb{R}^+$, $\left(\prod_{i=1}^n a_i \right)^{\frac{1}{n}} \leq \frac{1}{n} \sum_{i=1}^n a_i$."

Base Case: $n=2$.

Let $a_1, a_2 \in \mathbb{R}^+$ be arbitrary.

Then $a_1^2 - 2a_1a_2 + a_2^2 = (a_1 - a_2)^2 \geq 0$.

$$a_1^2 + a_2^2 \geq 2a_1a_2$$

Induction Step.

Let $n \in \mathbb{N}$ with $n \geq 2$

Assume $p(n)$.

Let a_1, \dots, a_{n-1}

$$\begin{cases} p(n) \text{ IMP. } p(n-1) \\ p(n) \text{ IMP. } p(2n) \end{cases} \Rightarrow \forall n \in \mathbb{N} \ p(n). \text{ WTF?}$$

In general

To prove $\forall i \in \{0, 1, \dots, n\}. p(i)$

Base case: $p(0)$

Induction Step: Let $i \in \{0, \dots, n-1\}$ be arbitrary.

Assume $p(i)$

\vdots
 $p(i+1)$

Hence $\forall i \in \{0, 1, \dots, n\}. p(i)$.

Strong (Complete) Induction

Suppose we know $\forall i \in \mathbb{N}. (j < i \text{ IMPLIES } p(j)) \text{ IMPLIES } p(i)$

Then I know $\forall i \in \mathbb{N} \ p(i)$

Template. Let $i \in \mathbb{N}$ be arbitrary.

Assume $\forall j \in \mathbb{N}. j < i \text{ IMPLIES } p(j)$

\vdots
(cases)

$p(i)$

$\forall j \in \mathbb{N}. (j < i) \text{ IMPLIES } p(j)$

direct proof

$\forall i \in \mathbb{N} \ \forall j \in \mathbb{N}. (j < i) \text{ IMPLIES } p(j)$

generalization

$\forall i \in \mathbb{N} \ p(i)$

strong induction

Alternative Template. (Deal with base separately)

Base case

Alternative Template. (Deal with base separately)

Base case

⋮
 $p(0)$

Induction Step.

Let $i \in \mathbb{N}^+$ be arbitrary.

Assume $\forall j \in \mathbb{N}. (j < i) \text{ IMPLIES } p(j)$.

⋮

$p(i)$

⋮

$\forall i \in \mathbb{N}. p(i)$

Strong induction.