

Lecture Mar 4

Monday, March 4, 2019 11:10 AM

Analysis of Algorithms

For any algorithm A and any input I to A, let

$$t_A(I) = \# \text{ of steps } A \text{ performs on } I$$

What is a step? Chosen to reflect the running time to within a constant factor

Ex Linear Search

LINEAR SEARCH (L, x)

$i \leftarrow 1$

while $i \leq \text{length}(L)$ do

if $L[i] = x$ then
return i

$i \leftarrow i + 1$

return 0

Define step: number of comparisons with x

On input $[2, 4, 6, 8]$

Let $T_A(n)$ represent the running time of input size n
worst case (otherwise not well defined)

Formally, $T_A(n) = \{t_A(I) \mid \text{size}(I) = n\}$

$T_A: N \rightarrow N$

whose expectation is taken over
a probability space of all inputs of
size n

Def: let $T_A'(n) = \text{average time complexity}$
 $= E[t_A]$

$$\text{When all input equally likely: } T_A'(n) = \frac{\sum \{t_A(I) \mid \text{size}(I) = n\}}{\#\{I \mid \text{size}(I) = n\}}$$

Straight line code (no loops, procedure calls, if statements) $O(1)$

If-statements

if C then A else B
 $\alpha(h)$ $\alpha(f)$ $\alpha(g)$

$O(h(n) + \max\{f(n), g(n)\})$

Procedure (and function)

Calls P , $T_p(m) \in O(f(m))$
If p is called with an input
of size $g(n)$ $O(f(g(n)))$ time

Ex. Merge Sort

MERGE(A, B, C)

$i \leftarrow 1$
 $j \leftarrow 1$
 $k \leftarrow 1$
while $i \leq \text{length}(A)$ and $j \leq \text{length}(B)$ do
if $A[i] \leq B[j]$
 $C[k] \leftarrow A[i]$
 $i \leftarrow i + 1$
 $k \leftarrow k + 1$

Analysis:

① Step: assignments to C

let $m = \text{length}(A)$, $n = \text{length}(B)$

$$t_{\text{merge}}(m, n) = m + n$$

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while i ≤ length(A) and j ≤ length(B) do
  if A[i] ≤ B[j]
    then C[k] = A[i]
    else C[k] = B[j]
    k ← k+1
  if i ≥ length(A)
    then while j ≤ length(B) do
      C[k] = B[j]
      j ← j+1
      k ← k+1
    else while i ≤ length(A) do
      C[k] = A[i]
      i ← i+1
      k ← k+1

```

$$t_{\text{merge}}(m, n) = m+n$$

② step: comparison between A[i] and B[j]

bad input (upper bound).

last elements of A, B have the
2 largest inputs.

$$t_{\text{merge}}(m, n) = m+n - 1$$

Need both upper and lower bound.
(depends on input)

To prove $T_A \leq u$, must show for all inputs I, $t_A(I) \leq u(n)$
(size n)

$T_A \geq l$, need to show for some input I, $t_A(I) \geq (ln)$

less than the biggest thing